Is the 1990's US Expansion Similar to the 1960's?

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Abstract

Statistical similarities among the latest long expansion in the U.S. and some other past expansions, in particular that of the 1960s, are examined. Corresponding to the definition of statistical similarity, a test based on the covariance matrices of business cycle component variables for the different expansions is proposed. Among available tests, the test based on partial common principal component analysis is argued to be most appropriate. The test is applied to the components of both GDP and the composite coincident index. As a result, the 1990s expansion is concluded to be statistically similar to that of the 1960s.

KEY WORDS: Business Cycle; Statistical Similarity; Covariance Matrix Structure; Composite Index; Partial Common Principal Component Analysis

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An interesting current topic concerning the US economy at the moment is how long the present long expansion, starting from 1991, will continue. Since the last trough in March 1991 the expansion has so far continued for nearly 100 months until mid 1999. In fact, the present expansion in the US economy is so long and strong that it has even produced a view that the US economy may have fundamentally changed and entered a new era of prolonged economic growth supported by high productivity. Many business cycle economists, however, think that the business cycle is still very much alive and believe the US economy will eventually, inevitably go into its next contraction.

The longest expansion after World War II occurred in the 1960s, beginning in March 1961 and ending in December 1969 for a total duration of 106 months. Because of its length and strength, the present expansion is sometimes compared with the 1960s expansion and one recent issue of discussion among business cycle economists and other analysts is whether or not the duration of the present expansion will exceed that of the 1960s expansion. In this comparison the duration of the two expansions is emphasised because they represent the two longest expansions since World War II and, in terms of this duration characteristic, these two expansions are now clearly quite similar.

However, there are several other important characteristics of business cycle phases other than duration, viz. amplitude, asymmetry, strength (or depth), the classification of types of phases, and so on. For example, the average annual growth rates of GDP components are sometimes used to characterise phases or states of the business cycle. Table 1 shows the average growth rates of the GDP components for all US post-war expansions. We could discuss similarities among expansions by looking at the similarity or dissimilarity of such growth rates across expansions.

Statistical tests, such as tests for equality of means, could then be used to judge whether the expansions were statistically similar or not.

Table 1. Average Annual Growth Rate (%) of the US GDP Components by Expansions

In a more sophisticated way, Pagan (1997) characterises business cycles in many countries in terms of duration and asymmetry using simple statistical models and simulation, and Hess and Iwata (1997) also analyse the business cycle features of duration and amplitude by a very similar method. In these recent studies, univariate time series models are applied to GDP and simulation is carried out to characterise business cycle features. King and Plosser (1994) and Simkins (1994) use a similar approach using real business cycle models. These authors use real GDP as the representative indicator of business cycle fluctuations. On the other hand, Layton (1998) and others have applied nonlinear time series models, viz. regime switching models developed by Hamilton (1989), to the coincident and leading composite indexes to investigate the characteristics of US business cycles.

As noted above, comparisons of business cycle phases often focus on the relative growth rates of GDP and its components or the components of the coincident composite index. However, in comparing different business cycle phases, it is not sufficient to frame the analysis solely on the basis of comparing such growth rates. In this paper, we will investigate the statistical similarity between the present expansion and other expansions in the U.S. in a somewhat different way. We examine the extent of overall statistical similarity among expansions using statistical tests based on the variance-covariance matrices (hereafter covariance matrices) of a vector of business cycle variables across different expansions. Expansions are regarded as statistically similar if the covariance matrices have a particular type of similarity.

The suggested approach provides a method to comprehensively assess the degree of phase similarity since the covariance matrix for each phase provides a useful statistical summary of the interrelationships among the business cycle component variables. However, we nonetheless recognise that comparing the average growth rates of business cycle component variables across phases is a common and very important aspect of the issue of similarity. Since our approach uses covariance matrices in the comparisons of phases, we focus on the covariation of variables around their means across phases rather than investigating the similarity or dissimilarity in the mean growth rates themselves. In this sense, our approach must be regarded as complementary to the more usual approach.

There are a couple of motivations, in general, for investigating the extent of similarities among different business cycle phases. Firstly, in characterising past phases, this may help improve our understanding of the nature of what happened in the economy during different economic episodes. Secondly, in terms of the current US expansion, identifying earlier expansions which were similar to it may help to determine what policy responses may be appropriate in the current episode.

In Section 1, the statistical similarity concept used in this paper is defined. Section 2 presents the statistical test used in the paper for examining statistical phase similarities. The test we adopt is based on the 'partial' common principal component analysis proposed by Flury (1987). The proposed method can be applied to both expansions and contractions or to whole cycles, but in this paper our interest is on post World War II US expansions only. We apply the test to the coincident composite index components in Section 3 and to the components of GDP in Section 4. Section 5 contains some concluding remarks.

1. Definition of Statistical similarity among Phases of Business Cycles

Before examining business cycle statistical similarities, we first have to define what we mean by similarity and, indeed, we must also even define what we mean by the business cycle. We will discuss statistical similarity from the point of view of principal component analysis (PCA) and common principal component analysis (CPCA) which generalises PCA to the case of more than one group. Applications of PCA and CPCA to the components of the Japanese diffusion index are seen in Kariya (1986) and Katsuura (1988), respectively, and the optimality of applying PCA to time series data is discussed in Kariya (1993). Also, Stock-Watson's model for business cycle analysis in Stock and Watson (1991, 1993) is an application of a type of dynamic principal component or factor analysis.

1.1 Definition of Business Cycles

An appropriate definition of the business cycle has been discussed by a number of authors. The most commonly used definition is to regard business cycles as significant cyclical fluctuations in 'aggregate economic activity' as defined by Burns and Mitchell (1946)[1]. By this definition, one approach is to define business cycle phases by analysing a group of selected related economic variables. Another approach is simply to use cyclical movements in GDP as a proxy for the business cycle. Using GDP, one widely used convenient rule of thumb to determine peaks of

[1] See Diebold and Rudebush (1996), King and Plosser (1994) and so on.

the business cycle is the period prior to that for which GDP falls for at least two consecutive quarters. An analogous rule is applied to identify trough dates. Some countries, for example Australia, use this simple rule to establish a business cycle chronology. Other countries, in particular the U.S., use a much more elaborate statistical approach involving a range of economic indicators to determine the business cycle chronology.

When cyclical fluctuations in GDP are analysed, the demand components of it, viz. consumption, private investment, government expenditure and net exports are often given much attention. If we adopt GDP as a proxy for the business cycle, it is natural then to regard business cycles as the combined fluctuations in these components. Therefore, irrespective of whether one prefers to use the composite coincident index or GDP, business cycles may be regarded as a weighted average of fluctuations in the relevant components. If we assume linearity, we could therefore express the business cycle as

$$f_t = \beta_1 x_{1t} + \beta_2 x_{2t} + \cdots + \beta_p x_{pt} \quad , \tag{1}$$

where f denotes the business cycle, x is a variable in the coincident index or GDP, β is the weight attached to the variable, and p is the number of variables. We use this general definition of the business cycle throughout this paper.

1.2 Definition of Similarities

Given the business cycle definition as expressed in (1) in this paper, statistical similarities or dissimilarities between business cycles phases are measured by the nature of the relationships among the various variables, i.e. if correlations among variables between two phases are statistically similar, those two phases may be regarded as statistically similar. For example, consider the situation in which consumption, as a GDP component, shows similar average growth between two phases. This suggests a certain similarity between the two phases but this is only part of the picture. We can not conclude whether these two phases are similar or not in respect of consumption unless we also consider the relation of consumption to other variables in the two episodes as well. A useful overall measure of the interrelationships among variables is their covariance or correlation matrix[2]. Therefore, a potentially useful measure of statistical similarity is the extent to which the covariance matrices of different phases are statistically the same. As noted earlier, however, for the purposes of this paper, our analysis of similarities abstracts from incorporating an explicit statistical comparison of component variable mean growth rates.

Whilst the above discussion is quite intuitive, we now formally define statistical similarity based on equation (1). Consider the question of statistical similarity between two phases, viz. phase 1 and 2. Analogous to equation (1) we would have:

$$f_t = \beta_1^{(1)} x_{1t} + \beta_2^{(1)} x_{2t} + \cdots + \beta_p^{(1)} x_{pt} \quad (t = \tau_1, \tau_1 + 1, \cdots, \tau_1 + T_1), \tag{2}$$

$$f_t = \beta_1^{(2)} x_{1t} + \beta_2^{(2)} x_{2t} + \cdots + \beta_p^{(2)} x_{pt} \quad (t = \tau_2, \tau_2 + 1, \cdots, \tau_2 + T_2) \tag{3}$$

(where the phases are denoted by the parenthesised superscripts). In these equations, the statistical similarity between the two phases would be judged by whether the corresponding coefficients in the two equations are statistically equal or not. This can be easily generalized to three or more phases.

In this discussion, however, there are two problems: the first is how to estimate the β coefficients, and the other is how to relate those expressions to the

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[2] Because a correlation matrix can be regarded as the covariance matrix of standardised data, we use the term covariance matrix and correlation matrix interchangeably. Also, see footnote 6.

covariance structure. To answer these questions simultaneously, (partial) common principal component analysis, explained in the next subsection, is useful.

1.3 Hierarchy of Similarities among Covariance Matrices

Flury (1988) defined a hierarchy of similarities among the structure of covariance matrices by the following five levels:

(i) equality ; $\Sigma_1 = \Sigma_2 = \cdots = \Sigma_k$

(ii) proportionality ; $\Sigma_i = \rho_i \Sigma_1$

(iii) common principal component model ; $\Sigma_i = \mathrm{B}\Lambda_i \mathrm{B}'$,

where $\mathrm{B} = \left(\vec{\beta}_1, \vec{\beta}_2, \cdots, \vec{\beta}_p\right)$ and $\Lambda_i = diag(\lambda_{i1}, \lambda_{i2}, \cdots, \lambda_{ip})$ with $\lambda_{ij} = \vec{\beta}_j' \Sigma_i \vec{\beta}_j$

and $\vec{\beta}_j$ s the common eigenvectors.

(iv) partial common principal component model ; $\Sigma_i = \mathrm{B}^{(i)} \Lambda_i \mathrm{B}^{(i)'}$,

where $\mathrm{B}^{(i)} = \left(\vec{\beta}_1, \cdots, \vec{\beta}_{q,} \vec{\beta}_{q+1}^{(i)}, \cdots, \vec{\beta}_p^{(i)}\right)$, $q \le p$

and the first q eigenvectors only are common.

(v) arbitrary,

where $\Sigma_i \; (i = 1, 2, \cdots, k)$ denotes the i th covariance matrix in k groups (phases), $\vec{\beta}$ is eigenvector of order p , B and $\mathrm{B}^{(i)}$ are orthogonal square matrices of order p , Λ_i is a diagonal square matrix of order p , and p is the number of variables. The strongest form of statistical similarity is (i), and the weakest is (iv).

The common principal component (CPC) model in stage (iii) is proposed by Flury (1984). Principal components are common across groups if there exists a common orthogonal matrix whose columns are eigenvectors which are common for all groups. Note that in the usual principal component analysis (PCA), which assumes stage (v) implicitly, $\mathrm{B}_i'\Sigma_i\mathrm{B}_i = \Lambda_i$, where Λ_i is diagonal with eigenvalues of Σ_i. In normal PCA with $k = 2$, we can express the principal components using original data as follows:

$$f_{jt} = \beta_{1j}^{(1)} x_{1t} + \beta_{2j}^{(1)} x_{2t} + \cdots + \beta_{pj}^{(1)} x_{pt} \quad (t = \tau_1, \tau_1 + 1, \cdots, \tau_1 + T_1;\ j = 1, \cdots, p), \tag{4}$$

$$f_{jt} = \beta_{1j}^{(2)} x_{1t} + \beta_{2j}^{(2)} x_{2t} + \cdots + \beta_{pj}^{(2)} x_{pt} \quad (t = \tau_2, \tau_2 + 1, \cdots, \tau_2 + T_2;\ j = 1, \cdots, p). \tag{5}$$

Also, using the property of orthogonality, (4) and (5) can be written in factorial form,

$$x_{it} = \beta_{i1}^{(1)} f_{1t} + \beta_{2}^{(1)} f_{2t} + \cdots + \beta_{ip}^{(1)} f_{pt}$$

$$x_{it} = \beta_{i1}^{(2)} f_{1t} + \beta_{i2}^{(2)} f_{2t} + \cdots + \beta_{ip}^{(2)} f_{pt}$$

Therefore, in the CPC model of level (iii), statistical similarity of covariance matrices is defined as

$$\beta_{ij}^{(1)} = \beta_{ij}^{(2)} = \beta_{ij} \, (i, j = 1, 2, \cdots, p), \tag{6}$$

by which (4) and (5) would be expressed without superscripts.

At this point it is useful to draw out the connection between the business cycle definition embodied in (1) and the notion of phase similarity. Phase similarity was defined as existing if the coefficients in (2)-(3) were statistically the same. This would be equivalent to equations (4)-(6) for the special case of $j = 1$ (i.e. one linear combination of p variables being invariant across the two episodes). However, (4)-(6) embody the very much stronger case where $j = p$ (i.e. all p possible independent linear combinations of the p variables being invariant across the two episodes).

The CPC model assumes p common eigenvectors. But, in practice, we do not need to require all p eigenvectors to be the same in analysing statistical similarities in business cycles phases. In other words, the definition of statistical similarity in (6) is far too strict. If Equations (2) and (3) represent the business cycle in the two episodes and the coefficients across these two equations are statistically the same, statistical similarity between the two business cycle phases may be regarded as existing. This situation corresponds to the case in which just one common eigenvector exists and this suggests that, for business cycle analysis purposes, partial common principal component analysis is the more relevant approach.

Given this, using partial CPC model described by level (iv) similarity, we can express a definition of statistical similarity as:

$$\beta_{ij}^{(1)} = \beta_{ij}^{(2)} = \beta_{ij} (i = 1,2,\cdots, p; j = 1,2,\cdots,q) . \tag{7}$$

Note that the difference between (6) and (7) is the range of j. And, using the expression involving the original data, we can express

$$f_{jt} = \beta_{1j} x_{1t} + \beta_{2j} x_{2t} + \cdots + \beta_{pj} x_{pt} \quad (j = 1,2,\cdots,q \text{ for all } t) , \tag{8}$$

$$f_{jt} = \beta_{1j}^{(1)} x_{1t} + \beta_{2j}^{(1)} x_{2t} + \cdots + \beta_{pj}^{(1)} x_{pt} \quad (j = q+1,\cdots, p, t = \tau_1, \tau_1 + 1,\cdots, \tau_1 + T_1)$$

$$f_{jt} = \beta_{1j}^{(2)} x_{1t} + \beta_{2j}^{(2)} x_{2t} + \cdots + \beta_{pj}^{(2)} x_{pt} \quad (j = q+1,\cdots, p, t = \tau_2, \tau_2 + 1,\cdots, \tau_2 + T_2) .$$

The differences between the CPC and partial CPC models may be further clarified by using the factorial form. In partial CPC model, we can express

$$x_{it} = \beta_{i1} f_{1t} + \cdots + \beta_{iq} f_{2t} + \beta_{i,q+1}^{(1)} f_{q+1,t} \cdots + \beta_{ip}^{(1)} f_{pt} \ ,$$

$$x_{it} = \beta_{i1} f_{1t} + \cdots + \beta_{iq} f_{2t} + \beta_{i,q+1}^{(2)} f_{q+1,t} \cdots + \beta_{ip}^{(2)} f_{pt} \ ,$$

where the first q elements are common in both periods.

If $q = 1$, (8) becomes equation (1) as a special case, and corresponds to (2) and (3) with the same coefficients. However, in investigating the statistical similarities among the phases of business cycles, q does not have to be equal to one. Indeed, the greater is q, the greater the degree of statistical similarity the two phases would have. This means that for phases of business cycles to be regarded as, in some sense, statistically similar, we require at least level (iv) similarity with $q=1$, and we believe this criteria might be useful for examining similarities of covariance matrices among business cycle phases. Of course, if CPCs exist it would be desirable if they accounted for a significant proportion of the total variance in the underlying component variables. When statistical similarity is obtained for just $q=1$, this would be analogous to the single index model adopted by Stock and Watson (1991, 1993) which in turn was theoretically based upon Sargent and Sims (1977).

2. Testing Similarities among Covariance Matrices

2.1 Testing Equality of Covariance Matrices[3]

To test the equality of k covariance matrices, corresponding to the strongest level of similarity as defined in (i), the null and alternative hypotheses are:

$$H_0 : \Sigma_1 = \Sigma_2 = \cdots = \Sigma_k$$

$$H_A : \Sigma_i \neq \Sigma_j \quad (i \neq j)\,.$$

The likelihood test statistics of above hypothesis is,

$$\lambda_1 = \frac{\prod_{i=1}^{k} |S_i|^{\frac{T_i}{2}}}{\left|\sum_{i=1}^{k} S_i\right|^{\frac{T}{2}}} \cdot \frac{T^{\frac{kT}{2}}}{\prod_{i=1}^{k} T_i^{\frac{kT_i}{2}}},$$

where S_i is i th sample covariance matrix, T_i is the number of observations in the the i th group (phase) and $T = \sum_{i=1}^{k} T_i$. Since the asymptotic null distribution of $-2\log\lambda_1$ is χ^2 distribution with $(k-1)p(p+1)/2$ degrees of freedom, the usual

[3] See Anderson (1984) pp.404-426.

procedure for testing the null hypothesis can be carried out. Some test statistics which adjust λ_1 have also been proposed.

However, in examining statistical similarities of business cycle phases defined by (7) or (6), we do not require strict equality of covariance matrices. Clearly, if covariance matrices are equal, identical eigenvectors are always obtained, but even if covariances matrices are not strictly equal, there nevertheless could be cases in which one or more (statistically) identical eigenvectors are obtained, viz. the case of the existence of (partial) CPC in the covariance matrices. As noted in the previous section, statistical similarity of phases defined in this paper requires only that at least one eigenvector exists which is statistically the same across alternative phases. See Katsuura (1997) for examples in which H_0 is rejected by the test for equality of covariance matrices, but statistically similar (statistically identical) eigenvectors are obtained for alternative sample periods in stock price data. Nevertheless, for completeness and for expositional purposes, in the next section we will also report the results of this test for the business cycle data which we analyse.

2.2 Testing Common Principal Components Hypothesis[4]

When testing the existence of CPC among k covariance matrices, the following is the null hypothesis to be tested:

$$H_{CPC} : \mathrm{B}'\Sigma_i \mathrm{B} = \Lambda_i$$

$$H_A : not\ H_{cpc} .$$

The test statistic for this hypothesis is,

[4] On testing stage (ii) of similarity, viz. proportionality of covariance matrices, see Flury (1988) chap. 5. We omit this issue in this paper because we are not interested in proportionality of covariance matrices in terms of business cycle analysis using growth rates of the component variables.

$$\chi^2_{CPC} = -2\log\frac{L(\hat{\Sigma}_1,\hat{\Sigma}_2,\cdots,\hat{\Sigma}_k)}{L(S_1,S_2,\cdots,S_k)}$$

$$= \sum_{i=1}^{k} n_i \log\frac{|diagF_i|}{|F_i|} = \sum_{i=1}^{k} n_i \log\frac{\prod_{j=1}^{p} f_{jj}^{(i)}}{\prod_{j=1}^{p} l_{ij}}, \qquad (9)$$

where $\hat{\Sigma}$ is the maximum likelihood estimate (MLE) of Σ in the CPC hypothesis, $F_i = \hat{B}' S_i \hat{B}$ and $\hat{B}$ is the maximum likelihood estimator of CPC. The asymptotic null distribution of χ^2_{CPC} is χ^2 distribution with $(k-1)p(p-1)/2$. For further details of the estimation and the statistical test in CPCA, see Flury (1984,1988).

If the null hypothesis can not be rejected, the CPC hypothesis is accepted, implying that, according to the above definition, similarities among phases are found to exist. Katsuura (1997) used this statistic for examining statistical similarities defined by (6) for the Japanese diffusion index components, but no business cycle statistical similarity was found[5].

2.3 Testing the partial Common Principal Components Hypothesis

In the previous section, we explained that statistical similarities among phases of business cycles are captured by (7) for at least $q=1$. Therefore, it is appropriate to test the existence of such similarity based on the partial CPC model (level (iv) similarity). Estimation and test statistics are similar to that of the CPC model. For further details of partial CPC concept, estimation and statistical test, see Flury (1987, 1988).

The partial CPC hypothesis and its alternative are

$$H_{CPC}(q): \mathrm{B}^{(i)}{}'\Sigma_i \mathrm{B}^{(i)} = \Lambda_i$$

$$H_A : not\ H_{cpc}(q),$$

and the test statistic for this hypothesis is

$$\chi^2_{CPC(q)} = -2\log\frac{L(\tilde{\Sigma}_1, \tilde{\Sigma}_2, \cdots, \tilde{\Sigma}_k)}{L(S_1, S_2, \cdots, S_k)} = \sum_{i=1}^{k} n_i \log\frac{\left|\tilde{\Sigma}_t\right|}{\left|S_i\right|}, \tag{10}$$

where ~ denotes the MLE of the partial CPC model assuming a particular value for q. The asymptotic distribution of $\chi^2_{CPC(q)}$ under the partial CPC hypothesis is χ^2 distribution with $(k-1)q(2p-q-1)/2$ degrees of freedom. The test procedure is the same as that of CPC except for the selection of q, the number of partial CPC. It is possible to determine the value of q in a step-wise fashion, for example, in accordance with the magnitude of diagonal elements of estimated Λ_i. Note that the cases of $q = p-1$ and $q = p$ are equivalent and correspond to the CPC case in the previous subsection.

MLE of $\mathrm{B}^{(i)}$, which includes both common and specific eigenvectors, provides information about the influence of individual variables on partial CPCs. As such, the identified variable may be expected to contribute significantly to business cycle fluctuations as expressed by (8). These estimated eigenvectors will therefore be useful in interpreting the characteristics of the phases as well as those of the obtained principal components.

In summary, we use the test statistic (10) to examine statistical similarities among business cycle phases as defined in (7).

[5] Katsuura (1997) uses levels data - not transformed into growth rates - to test similarities. Therefore,

3. Empirical Results Using the Coincident Index Components

In this section, we examine statistical similarities between the present and past US expansions using the ECRI (Economic Cycle Research Institute) coincident index components.

The dates of the phases, i.e., contractions and expansions, are determined in the U.S. by the NBER (National Bureau of Economic Research). The official chronology and durations of phases are shown in Table 2 on both a monthly and quarterly basis. The empirical analyses in the present and following sections are based on these chronologies. Available data are categorised according to the expansion dates, and, for each expansion, the relevant sample covariance matrix is calculated[6].

Table 2. Monthly and Quarterly Chronology of Business Cycles in the U.S.

3.1 Data

As discussed in Section 1, a widely accepted definition of empirical business cycles employs the components of a coincident composite index to determine a phase chronology. Therefore, we use these variables to examine statistical similarities among phases in this section. One advantage is that most of these variables are available monthly which allows us to use a greater number of observations than in the case of quarterly data as in the following section. Another advantage in using such

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we can not directly compare this earlier work with the results in the following chapter.

[6] It is possible to apply PCA or CPCA to correlation matrices. The use of correlation matrices is often effective when the variables are measured in a variety of units. However, as we transform the data into growth rates, the problem of different units does not exist here. If we had used correlation matrices, different results would have been obtained. We also felt that for business cycle analysis it was not appropriate to standardise the variance of component variables to 1.

variables is that the definition of statistical similarity adopted here, based on (7) and (8), is somewhat analogous to the approach for constructing business cycle composite indices as linear combinations of the component variables[7].

The variables used in this section are the five components of the ECRI coincident composite index. Although the ECRI coincident composite index consists of six components, we use only the components with monthly observations and the sole quarterly based component (real GDP) is excluded. The components used here are then as follows;

Personal income (C1)

Industrial production (C2)

Manufacturing and trade sales (C3)

Unemployment rate (C4)

Non-farm employees (C5).

The available data spanned January 1959 to January 1999 and the subsample periods are determined by the monthly chronology shown in Table 2. However, since the distribution of the test statistic in (10) requires a large number of observations, we only use the expansions whose durations are greater than thirty months[8]. As a result, we analyse five expansions, viz., E4 (March 1961 - December 1969), E5 (December 1970 – November 1973), E6 (April 1975 – January 1980), E8 (December 1982 – July 1990) and the present expansion, E9 (April 1991 -) [9]. The components are transformed into month-to-month growth rates except for C4. Since C4, unemployment rate, is already expressed as a ratio, it is transformed into month-to-

[7] Of course, the calculation of the composite index is not as straightforward as in (8).

[8] In order to use as many as possible of the same expansions used in the quarterly analysis in the following section, we adopt in that section the criteria of incorporating expansions with more than twenty quarters. Nevertheless, the outcome is that we are able only to analyse four expansions in the quarterly analysis as compared with five expansions in the monthly analysis.

[9] The figures after E (Expansion) correspond to the orders of the post war expansions shown in Table 2.

month differences. By transforming the data into growth rates and differences we should ensure the resulting data have the property of stationarity. In estimating the covariance matrix and PCs, stationarity is an important property. See Kariya (1993) on the role of the stationarity in PCA.

3.2 Results Using Coincident Index Components

Using the calculated covariance matrices for each expansion, the results of a standard PCA for each expansion are shown in Table 3. The principal components (PCs) are ordered according to the magnitude of eigenvalues in the tables. Interpretation of the components should be carried out in a careful way by looking at the relative magnitudes of the elements of the eigenvectors. In particular, we need to pay special attention to the largest eigenvector element. For example, for the expansions in the 1960s (E4) and 1990s (E9), the first PC in E4 corresponds to the second PC in E9 because the largest element in both of these eigenvectors corresponds to the same underlying variable, viz. C3. In general, C3, manufacturing and trade sales, has the greatest value in the first PC for each expansion except in the case of E9 where it is observed to be the largest element in the second PC. C2, industrial production, has the second largest weight in the first PC for all expansions except E9 where it has the second largest weight in the second PC. If we interpret these PCs as indicative of the business cycle, fluctuations in C3 and C2 are relatively more important. This interpretation, in which business cycle fluctuations are captured mainly by variation in industrial production and manufacturing and trade sales, seems to be intuitive and reasonable.

The period used in E9, of course, does not include the whole expansion. This may be the cause of the different pattern of the elements of eigenvectors in the PCs.

Alternatively, we could conclude that the core features of the present expansion are slightly different to those of the other expansions.

Table 3. Results of PCA for Each Expansion for Coincident Composite Index Components

We calculate the test statistic (10) for examining statistical similarity of expansions for different values of q in Table 4. In selecting eigenvectors for a value of q, we extract and define the j th partial CPC according to the j th largest sum of corresponding diagonal elements in CPCA, viz. $\sum_{i=1}^{k} f_{jj}^{(i)}$ in (9), because the estimators and algorithm of partial CPCA are based on CPCA. Moreover, the values of q are increased in step-wise fashion from one to p. Recall that if $q = p$ or $p-1$, it corresponds to the CPC model.

Calculated χ^2 values for test statistic (10) between E9 and other expansions for each value of q are shown in Table 4. A significant calculated value means that the partial CPC hypothesis, $H_{CPC}(q)$, is rejected, which implies no statistical similarity exists between the relevant expansions. As we have defined statistical similarity among phases as the existence of at least one partial CPC, the expansion in the late 1970s, E6, is not statistically similar to the present expansion, E9. However, the other expansions, E4, E5 and E8 can be said to be statistically similar to E9. Looking at the value of q, the maximum possible value of $q = 5$ is obtained in testing for statistical similarity between both E9 and E4 and E9 and E5. This corresponds to level (iii) similarity in Section 2, implying the statistical similarity between the 1990s expansion and that of the 1960s and early 1970s expansions is very strong. Statistical similarity between the present and the immediately previous expansion also exists, but it is not as strong. Also, to emphasise the advantages of our proposed definition of similarity, the test statistics, $-2\log\lambda_1$, for the strict statistical equality of covariance matrices, viz. level (i) similarity, are reported in the last column of Table 4. Using these statistics, no similarities (equalities) are accepted which would imply that level (i) similarity is absent as far as the business cycle expansions under study are concerned. However, as we argued earlier, level (i) similarity – strict equality of covariance matrices of different phases - is far too strict a definition of similarity than is required for business cycle analysis purposes.

These similarities or dissimilarities between expansions may be at least partially explained by the general economic situations prevailing during the expansions in question. For example, the expansion in the late 1970s (E6) began after the previous deep contraction caused by the first oil crisis occurring at the end of 1973. The US economy subsequently recovered very rapidly from the deep trough in March 1975. We can see a similar situation in the expansion in the 1980s (E8) after the second oil crisis and very tight monetary policy in the early 1980s brought about the very pronounced contraction in 1981/82. Again, the US economy recovered very rapidly from the trough in November 1982. For example, looking at the unemployment rate (C4) as one indicator of the speed and strength of employment growth in each expansion, it was decreasing from the very earliest months of each of these two expansions and continued to fall throughout almost the entire duration of the expansions.

However, as the contraction preceding the present expansion was not as deep or as protracted as the above contractions, during its early months, the rate of growth in the economy was considerably more modest. Looking at the unemployment rate again, it continued to increase for some 20 months after the trough in March 1991 until November 1992, and only then did it begin to decline. In the case of the 1960s expansion (E4), although the economy expanded in the early part of the expansion in a similar way to E6 and E8 with unemployment decreasing from the trough, the economy did not always expand sufficiently rapidly throughout the entire expansion; the unemployment rate increased from the beginning of 1964 to mid 1965, then began to decline again[10]. These unique features of the expansions in question may explain the strong statistical similarities between both E9 and E4 and E9 and E5, the weaker similarity between E9 and E8, and the absence of any similarity between E9 and E6.

We should note here in passing that transitivity does not always hold in this type of analysis; for example, the χ^2 statistic relating to E6 and E8 is 7.517 for $q = 1$ (not shown in Table 4), and similarities between E6 and E8 and between E8 and E9 exist, but no statistical similarity exists between E6 and E9. This is not unusual because of the possibility of the existence of different sorts of pairwise statistical similarities.

Now, looking at the CPC results for E4 (the 1960s) and E9 (1990s) in Table 5, the first CPC seems to be a mixed vector of the first PC in E4 and the second PC in E9 because the coefficient of C3 is much greater than those of the other variables. Furthermore, recall that this first CPC by definition accounts for the largest proportion of aggregate variation across both phases (even though in the case of E9 the percentage variation explained is only .39). Given this, and the fact that it is statistically significantly common, the implication is that it is important as a

[10] The discussion of the unemployment rate or the strength of recovery is consistent with the results obtained in Layton and Smith (1999). They categorised expansions into two types, viz. a "slow or normal growth" expansion and a "fast growth" expansion. For example, the expansion in 1980s (E8) began as a “fast growth” expansion and then switched into a “normal or slow growth” expansion, whilst the 1990s recovery (E9) was categorised as a “slow or normal growth” expansion for the first

representation of the US business cycle. In this representation C3, manufacturing and trade sales, is found to be an important underlying component variable, not only for the first CPC of the E4 - E9 comparison, but also for the first CPC for each of the other two pairwise comparisons (see Table 5).

Table 4. χ^2 Statistics for Testing Similarities for Coincident Composite Index Components

Table 5. Estimated (Partial) CPC for Coincident Composite Index Components

4. Empirical Results for the GDP Components

4.1 Data

In this section, we use the GDP components to examine statistical similarities among expansions. As GDP and its components are published quarterly, we refer to the quarterly chronology presented in Panel b of Table 2. We use four components of GDP in the analysis, viz. private final consumption expenditure (C), gross domestic capital formation (I), government final consumption expenditure (G) and export (X). Although it is possible to use net exports (exports minus imports) as the fourth variable, it often takes negative values since the 1980s and the variability in its growth rate is extremely high. We could also use imports as the fifth variable. However, as it does not represent products produced in the economy it is difficult to interpret how it is directly relevant as far as the business cycle is concerned. Therefore, we use the above four GDP components only.

few years. Such characterisations of these two expansions are consistent with the weaker similarity

As mentioned in the previous section, we investigate only the expansions whose periods are at least twenty quarters. According to this criterion, four expansions, viz. E4 (1961:Q1 – 1969:Q4), E6 (1975:Q2 – 1980:Q1), E8 (1982:Q4 – 1990:Q3) and the present expansion, E9 (1991:Q2 -) are used in this analysis. The duration of the expansion in the early 1970s, which was found to have statistical similarity to the present expansion in the previous section, is too short to examine in this quarterly analysis. Though the expansion in the late 1970s, E6, also comprises a relatively small number of observations (20 quarters), it is the only expansion which was not statistically similar to the present expansion in the case of the coincident composite index components and we therefore include it to further clarify the situation. As before, the data for calculating covariance matrices are transformed into quarter-to-quarter growth rates.

4.2 Results for GDP Components

Analogous results for the US GDP components are shown in Tables 6-8. From Table 6, the first PC in the 1960s expansion seems to reflect export variation. For the other three expansions the first PC is interpreted as representing investment fluctuations, while the second PC is related mainly to export fluctuations. Intuitively this is because the variability in investment and exports is usually much greater than that of consumption or government expenditure. Together, these first two PCs account for about 95% of total variation in all four components for each of the expansions under study.

Calculated χ^2 test statistics in Table 7 imply that the present expansion is statistically similar to the expansions in the 1960s and 1980s but is again statistically

found in this analysis.

dissimilar to the expansion in the late 1970s. Whilst these results are broadly consistent with those in the previous section including the results of the test for strict statistical equality between covariance matrices, there are some interesting differences. One issue is the fact that, in Table 7, the null hypothesis of CPC is rejected for $q = 1$ but paradoxically is accepted for $q = 2$ and 3. This is logically inconsistent and is most likely explained by the relatively small sample size available for E6 (viz. 20 quarters). A second issue is that the statistical similarity between E8 and E9 is now found to be much stronger than in the case of the coincident composite index components analysis.

Each of the E4 - E9 and E8 - E9 comparisons shows strong similarity and corresponds to level (iii) similarity. The estimated CPCs are presented in Table 8. For E4 and E9, the first CPC consists of the first PC in E4 and the second PC in E9 which seems to represent export variation, and the second CPC is again interpreted as representing investment variation. For E8 and E9, the first CPC can be interpreted as loading heavily on the investment variable for both expansions, while the second CPC mainly reflects export variation. Again, this is quite consistent with the commonly held view that cyclical fluctuations in the economy are the result mainly of investment and foreign trade fluctuations.

As noted above, an interesting issue pertains to the relatively stronger similarity found in this analysis between E8 and E9 compared with that in the previous section. The stronger similarity is most likely caused by the different definition of the business cycle implicit in the analyses in the two sections. In the previous section, although GDP is not included because it is a quarterly series, it is certainly regarded as one indicator of the business cycle, but only one of several. The finding that E8 is very similar to E9 in this section is obtained only in terms of the components of this one single indicator of the business cycle. Therefore, the result is not inconsistent with the analysis of the composite index components in which the statistical similarity between E8 and E9 was found to exist but was somewhat weaker. Importantly, these components include employment measures which are considered by many business cycle analysts to be extremely important aspects of the business cycle. We have already discussed the fact that the patterns of employment variation were quite different between the E8 and E9 expansions. Obviously this differential employment aspect is not captured in the GDP components analysis. Furthermore, another reason for the stronger similarity using the GDP components is due to growth in investment (whose weight is the largest in the first CPC) increasing relatively rapidly during the early period of both expansions and then, in both cases, its growth moderating considerably in the later periods of the expansions.

Table 6. Results of PCA for Each Expansion for GDP Components

Table 7. χ^2 Statistics for Testing Similarities for GDP Components

Table 8. Estimated CPC for GDP Components

5. Conclusion

In this paper we have proposed a new method for examining the existence and strength of business cycle phase similarity. The method uses the framework of partial common principal component analysis and tests for the existence of common eigenvectors in the covariance matrices of business cycle component variables across different business cycle phases. We applied the method to the US coincident composite index components as well as to the components of GDP in order to examine the statistical similarities between the present long expansion of the 1990s and earlier expansions, particularly that of the 1960s. In the present analysis we only examined pairwise statistical similarities but it is straightforward to extend the analysis to consider several phases simultaneously.

It is sometimes argued that the expansions in the 1960s and the 1990s are similar because of their long durations. Duration, however, is only one aspect of an expansion and there are other dimensions of the issue of similarity which desirably should be examined. This paper focuses on statistical similarities of the covariance matrices of selected variables from different expansions. As a result, we found statistical similarity between the expansions in the 1960s and the 1990s in terms of the components of both GDP and the coincident composite index. Furthermore, the statistical similarity observed in the two expansions may be said to be quite strong owing to the fact that the level of observed similarity corresponds to a relatively high level in the hierarchy of covariance matrix similarity. The definition of statistical similarity used here, at a minimum, requires the existence of at least one partial CPC, viz. level (iv) similarity among covariance matrices.

We also found that the present expansion is statistically similar to expansions in the early 1970s (only for the components of coincident composite index) and the 1980s, but not that of the late 1970s. We speculate that, in the latter case, the lack of similarity may be due to the quite unique supply side impacts of the first major oil crisis in the mid 1970s which brought about the contraction which immediately preceded the expansion of the later 1970s. Quite similar results are obtained for the GDP components and the coincident composite index components for the other three expansions. More stable results, however, were obtained for both sets of components in the comparison of the expansions of the 1990s and 1960s and this provides quite strong evidence of statistical similarity between the two expansions. However, the present expansion has not yet finished. When we include data for the whole expansion, the results may well be changed and, indeed, certain aspects of the similarity may be strengthened, e.g. in relation to the proportion of variation explained by the CPCs, the statistical similarity of the eigenvectors, etc.

However, there are some problems with the approach suggested in this paper. First, as we have to separate the data according to the business cycle chronology, some phases include only a small number of observations; especially if we use quarterly data. This can cause some results to be very imprecise. Moreover, the method is based on ML estimation which assumes large samples. If we were to try to examine the similarities between contractions or between contractions and expansions, this problem would become even more serious[11]. A possible approach to this problem is to apply the least squares method developed by Clarkson (1988). However, the statistical properties of the least squares estimator is not as yet known.

Secondly, we have only used the contemporaneous covariances, but not the autocovariances. It would be possible to include lagged variables when calculating covariance matrices and this is a possible generalization of the method. Thirdly, only the covariance or correlation between any two variables is used. Using partial covariance or correlation might be another possible generalization.

Another important problem is that the results will be affected very much by the component variables selected. This, in turn, is related to the definition of the business cycle which one employs. Moreover, recall that the use of covariance matrices does not utilise potentially important information concerning phase similarity which could be inferred from an investigation of differences in component

[11] On the results for contractions, see Katsuura and Layton (1999).

variable mean growth rates across phases. The method does not incorporate such considerations but, instead, investigates the degree of similarity in the patterns of covariation among the component variables around their respective means across different phases. Despite these concerns, we believe the method as demonstrated in this paper could be a useful additional tool in business cycle analyses.

Table 1. Average Growth Rate (%) of the US GDP Components by Expansion

	start		end	quarters	GDP	C	I	G	X	M
E1	50:Q1	-	53:Q3	15	6.81	3.87	9.82	16.18	3.83	11.67
E2	54:Q3	-	57:Q3	13	3.91	4.23	7.49	0.04	9.19	5.59
E3	58:Q3	-	60:Q2	8	5.78	4.95	14.18	2.46	10.50	7.35
E4	61:Q1	-	69:Q4	35	4.80	4.84	7.16	3.77	6.14	9.15
E5	71:Q1	-	73:Q4	12	5.21	4.73	13.76	-0.69	10.79	5.91
E6	75:Q2	-	80:Q1	20	4.24	3.86	9.85	1.57	7.29	7.95
E7	80:Q4	-	81:Q3	4	4.18	2.01	23.44	0.97	-0.42	8.85
E8	83:Q1	-	90:Q3	31	3.75	3.70	5.31	3.22	8.70	9.13
E9	91:Q2	-	(98:Q4)	(31)	3.13	3.27	8.53	0.49	7.61	10.14

Note:
Figures are transformed into annual growth rates.
(98:Q4) represents the end of our available data and is not part of the official chronology.

Table 2. Monthly and Quartery Chronology of Business Cycles in the U.S.

a. Monthly						**b.quarterly**				
Trough	Peak	expansion	contraction	total		Trough	Peak	expansion	contraction	total
	Nov-48		11				48:Q4		4	
Oct-49	Jul-53	45	10	55		49:Q4	53:Q3	15	3	18
May-54	Aug-57	39	8	47		54:Q2	57:Q3	13	3	16
Apr-58	Apr-60	24	10	34		58:Q2	60:Q2	8	3	11
Feb-61	Dec-69	106*	11	11		61:Q1	69:Q4	35*	4	4
Nov-70	Nov-73	36*	16	16		70:Q4	73:Q4	12	5	17
Mar-75	Jan-80	58*	6	6		75:Q1	80:Q1	20*	2	2
Jul-80	Jul-81	11	16	27		80:Q3	81:Q3	4	5	9
Nov-82	Jul-90	92*	8	8		82:Q4	90:Q3	31*	2	2
Mar-91	(Jan-99)	94*				91:Q1	(98:Q4)	31*		

Date and quarters in parenthesis are not the official chronology, but the end of the data.
Figures are number of months or quarters of phases and whole cycles.
* denotes the phases used in this analysis.

Table 3. Results of PCA for Each Expansion for Coincident Composite Index Components

a.E4 (Mar.61 - Dec.69)							**b.E5 (Dec.70 - Nov.73)**					
	1st PC	2nd PC	3rd PC	4th PC	5th PC			1st PC	2nd PC	3rd PC	4th PC	5th PC
C1	0.0437	0.1370	0.9758	-0.1353	-0.0941		C1	0.2473	0.0462	-0.0917	0.2200	0.9380
C2	0.4040	0.8994	-0.1540	-0.0647	-0.0071		C2	0.3848	-0.1294	-0.0825	0.8578	-0.3043
C3	0.9115	-0.4097	0.0107	-0.0264	-0.0237		C3	0.8868	-0.0291	0.0453	-0.4419	-0.1243
C4	0.0129	-0.0080	0.0559	-0.2697	0.9612		C4	0.0034	-0.1685	0.9777	0.0957	0.0805
C5	0.0627	0.0671	0.1446	0.9508	0.2581		C5	0.0664	0.9757	0.1636	0.1067	-0.0745
eigenvalue	1.1106	0.3784	0.1220	0.0272	0.0421		eigenvalue	1.3331	0.0179	0.0539	-0.3309	0.2827
proportion	0.6609	0.2252	0.0726	0.0162	0.0251		proportion	0.6604	0.0089	0.0267	0.1639	0.1401
cum.prop.	0.6609	0.8861	0.9587	0.9749	1.0000		cum.prop.	0.6604	0.6693	0.6960	0.8599	1.0000
c.E6 (Apr.75 - Jan.80)							**d.E8 (Dec.82 - Jul.90)**					
	1st PC	2nd PC	3rd PC	4th PC	5th PC			1st PC	2nd PC	3rd PC	4th PC	5th PC
C1	0.1520	-0.0145	-0.0786	0.0635	0.9831		C1	0.0249	0.0572	0.9651	-0.2468	0.0610
C2	0.3854	-0.9159	-0.0663	-0.0511	-0.0750		C2	0.3383	0.9327	-0.0865	-0.0759	0.0490
C3	0.9064	0.3970	-0.0399	-0.0306	-0.1355		C3	0.9389	-0.3435	-0.0097	-0.0218	0.0038
C4	-0.0106	-0.0018	-0.5616	0.8217	-0.0964		C4	-0.0247	-0.0525	-0.0681	-0.0350	0.9954
C5	0.0826	-0.0573	0.8200	0.5633	0.0155		C5	0.0533	0.0783	0.2373	0.9652	0.0557
eigenvalue	1.4736	0.3536	0.0272	0.0168	0.1194		eigenvalue	1.2400	0.3079	0.1212	0.0192	0.0267
proportion	0.7403	0.1776	0.0137	0.0085	0.0600		proportion	0.7231	0.1795	0.0707	0.0112	0.0155
cum.prop.	0.7403	0.9179	0.9316	0.9400	1.0000		cum.prop.	0.7231	0.9026	0.9732	0.9845	1.0000

Table 3. Results of PCA for Each Expansion for Coincident Composite Index Components
(Cont'd)

d.E9 (Apr.91 - Jan.99)												
	1st PC	2nd PC	3rd PC	4th PC	5th PC							
C1	0.9561	-0.2925	-0.0183	-0.0066	0.0067							
C2	0.0841	0.2158	0.9679	-0.0916	-0.0343							
C3	0.2806	0.9314	-0.2320	0.0036	-0.0071							
C4	0.0022	0.0195	0.0557	0.2770	0.9591							
C5	0.0129	0.0095	0.0773	0.9565	-0.2809							
eigenvalue	0.7163	0.5609	0.1609	0.0110	0.0207							
proportion	0.4873	0.3816	0.1095	0.0075	0.0141							
cum.prop.	0.4873	0.8689	0.9784	0.9859	1.0000							

Table 4. c2 Statistics for Testing Similarities for Coincident Composite Index Components

Expansions	q=1		q=2		q=3		q=4(q=5)		accepted number of CPC	-2log(l1)
E4 - E9	5.9544		13.5628		14.7493		18.3027		5	553.5911
E5 - E9	5.0328		7.9354		9.1759		12.3227		5	420.5473
E6 - E9	11.6799	_	18.0128	_	19.5802	_	20.2862	_	0	436.5553
E8 - E9	6.6287		26.2592	_	29.9854	_	29.9879	_	1	520.1603
$\chi^2_{0.05}(\nu)$	9.4877		14.0671		16.9190		18.3070			24.9958

Note:
* denotes the finding that the partial CPC hypothesis or, in the case of column seven, the hypothesis of strict covariance matrix equality, is rejected at 5% significance level.
Column six is the maximum accepted number of significant partial CPC (q).
Column seven presents the test statistics for testing for the existence of level (i) similarity, viz. the strict equality of the covariance matrices of the phases.

Table 5. Estimated (Partial) CPC for Coincident Composite Index Components

a. E4 and E9 (q=5)							**b. E5 and E9 (q=5)**					
	1st CPC	2nd CPC	3rd CPC	4th CPC	5th CPC			1st CPC	2nd CPC	3rd CPC	4th CPC	5th CPC
C1	0.0347	0.9981	0.0485	0.0050	-0.0122		C1	0.2402	0.9704	-0.0227	0.0016	-0.0063
C2	0.3135	-0.0578	0.9438	-0.0040	-0.0869		C2	0.2762	-0.0466	0.9529	-0.0507	-0.1048
C3	0.9482	-0.0177	-0.3170	-0.0078	-0.0101		C3	0.9300	-0.2368	-0.2812	0.0015	-0.0011
C4	0.0156	-0.0039	0.0165	0.9809	0.1933		C4	0.0146	-0.0035	0.0563	0.9959	0.0698
C5	0.0351	0.0079	0.0780	-0.1944	0.9771		C5	0.0307	0.0012	0.0963	-0.0754	0.9920
E4							E5					
diagonal	1.1028	0.1221	0.3832	0.0394	0.0328		diagonal	1.3177	0.2834	0.3407	0.0554	0.0213
proportion	0.6563	0.0727	0.2281	0.0235	0.0195		proportion	0.7842	0.1687	0.2028	0.0329	0.0127
cum.prop.	0.6563	0.7290	0.9570	0.9805	1.0000		cum.prop.	0.7842	0.9528	1.1556	1.1886	1.2012
E9							E9					
diagonal	0.5743	0.6976	0.1659	0.0209	0.0112		diagonal	0.6009	0.6751	0.1620	0.0204	0.0116
proportion	0.3907	0.4746	0.1129	0.0142	0.0076		proportion	0.4088	0.4593	0.1102	0.0138	0.0079
cum.prop.	0.3907	0.8653	0.9782	0.9924	1.0000		cum.prop.	0.4088	0.8681	0.9783	0.9921	1.0000

Table 5. Estimated (Partial) CPC for Coincident Composite Index Components (Cont'd)

c. E8 and E9 (q=1)												
	common				Specific PC							
	PC		E8				E9					
	1st CPC	2nd PC	3rd PC	4th PC	5th PC	2nd PC	3rd PC	4th PC	5th PC			
C1	0.0225	0.0586	0.9650	-0.2452	0.0688	0.9996	-0.0144	-0.0061	0.0052			
C2	0.2912	0.9487	-0.0864	-0.0726	0.0495	0.0066	0.9517	-0.0958	-0.0169			
C3	0.9557	-0.2940	-0.0059	-0.0083	-0.0094	-0.0259	-0.2922	-0.0038	0.0240			
C4	-0.0079	-0.0562	-0.0688	-0.0053	0.9960	-0.0023	0.0488	0.2919	0.9552			
C5	0.0357	0.0833	0.2379	0.9667	0.0265	0.0077	0.0796	0.9516	-0.2946			
E4												
diagonal	1.2370	0.3104	0.1212	0.0195	0.0269							
proportion	0.7213	0.1810	0.0707	0.0114	0.0157							
cum.prop.	0.7213	0.9023	0.9730	0.9843	1.0000							
E9												
diagonal	0.5741					0.7009	0.1625	0.0110	0.0214			
proportion	0.3906					0.4768	0.1106	0.0075	0.0145			
cum.prop.	0.3906					0.8674	0.9779	0.9855	1.0000			

Note:
The results are only shown for the maximum number of q in Table 4.
"Diagonal" denotes diagonal elements of estimated Li in (partial) CPC model.

Table 6. Results of PCA for Each Expansion for GDP Components

a.E4 (61:Q1 - 69:Q4)						**b.E6 (75:Q2 - 80:Q1)**				
	1st PC	2nd PC	3rd PC	4th PC			1st PC	2nd PC	3rd PC	4th PC
C	-0.0098	0.0189	0.0661	0.9976		C	0.0508	-0.0440	-0.3145	0.9469
I	-0.3270	0.9447	0.0116	-0.0219		I	0.9351	-0.3453	-0.0268	-0.0751
G	0.0134	-0.0091	0.9977	-0.0658		G	0.1022	0.1365	0.9349	0.3113
X	0.9449	0.3273	-0.0095	0.0037		X	0.3355	0.9275	-0.1625	-0.0289
eigenvalue	38.2685	11.0777	1.4927	0.4014		eigenvalue	15.5950	9.7270	0.3899	0.3022
proportion	0.7468	0.2162	0.0291	0.0078		proportion	0.5995	0.3739	0.0150	0.0116
cum.prop.	0.7468	0.9630	0.9922	1.0000		cum.prop.	0.5995	0.9734	0.9884	1.0000
c.E8 (83:Q1 - 90:Q3)						**d.E9 (91:Q2 - 98:Q4)**				
	1st PC	2nd PC	3rd PC	4th PC			1st PC	2nd PC	3rd PC	4th PC
C	0.0017	-0.0558	0.1589	0.9857		C	-0.0145	-0.0620	0.9574	0.2818
I	0.9992	-0.0042	0.0394	-0.0083		I	0.9103	0.3929	0.0009	0.1304
G	-0.0403	-0.2031	0.9640	-0.1668		G	-0.1284	-0.0173	-0.2828	0.9504
X	-0.0040	0.9776	0.2095	0.0216		X	-0.3933	0.9173	0.0590	-0.0189
eigenvalue	17.0645	2.8108	0.8175	0.2699		eigenvalue	5.4873	4.3225	0.1207	0.5405
proportion	0.8140	0.1341	0.0390	0.0129		proportion	0.5240	0.4128	0.0115	0.0516
cum.prop.	0.8140	0.9481	0.9871	1.0000		cum.prop.	0.5240	0.9369	0.9484	1.0000

Table 7. c2 Statistics for Testing Similarities for GDP Components

Expansions	q=1		q=2		q=3(q=4)		accepted number of CPC	-2log(l1)	
E4 - E9	2.2678		4.8782		6.3809		4	141.4050	*
E6 - E9	9.3463	_	9.7430		9.8909		0	115.1431	*
E8 - E9	1.8811		4.4749		5.0003		4	109.0858	*
$\chi^2_{0.05}(\nu)$	7.8147		11.0705		12.5916			18.3070	

Note: See notes in Table 4.

Table 8. Estimated CPC for GDP Components

a. E4 and E9 (q=4)						**c. E8 and E9 (q=4)**				
	1st CPC	2nd CPC	3rd CPC	4th CPC			1st CPC	2nd CPC	3rd CPC	4th CPC
C	-0.0164	-0.0192	0.1899	0.9815		C	-0.0118	-0.0513	0.2373	0.9700
I	-0.3247	0.9432	0.0706	-0.0006		I	0.9977	0.0062	0.0679	-0.0042
G	0.0275	-0.0640	0.9793	-0.1903		G	-0.0669	-0.0063	0.9689	-0.2382
X	0.9453	0.3255	-0.0009	0.0224		X	-0.0072	0.9986	0.0179	0.0483
E4						E8				
diagonal	38.2594	11.0337	1.5176	0.4295		diagonal	17.0497	2.7329	0.9026	0.2774
proportion	0.7467	0.2153	0.0296	0.0084		proportion	0.8133	0.1304	0.0431	0.0132
cum.prop.	0.7467	0.9620	0.9916	1.0000		cum.prop.	0.8133	0.9437	0.9868	1.0000
E9						E9				
diagonal	4.8364	4.9474	0.5569	0.1303		diagonal	5.2925	4.4895	0.5665	0.1225
proportion	0.4619	0.4725	0.0532	0.0124		proportion	0.5054	0.4288	0.0541	0.0117
cum.prop.	0.4619	0.9344	0.9876	1.0000		cum.prop.	0.5054	0.9342	0.9883	1.0000

Note:
See notes in Table 5.

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